Age Patterns of Marital Fertility: Revising the Coale–Trussell Method

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This article revises the Coale–Trussell method for analyzing data from the World Fertility Survey by proposing and testing alternative log-linear and log-multiplicative models. The models, in one form or another, represent the structural constraint underlying the Coale–Trussell method on the variation in the age pattern of human fertility. With a Poisson distribution assumption for the number of births, several parameters of the models are simultaneously estimated via maximum likelihood. It is shown that the new approach can be adopted whenever fertility limitation is compared across multiple populations or subpopulations. Future users of the Coale–Trussell method for single populations or subpopulations are advised to use the new v_a estimates from this study in place of those of Coale and Trussell.

KEY WORDS: Age pattern; Birth control; Comparative study; Fertility; Log-multiplicative model; Measurement; Poisson regression.

1. INTRODUCTION

Let the observed marital fertility rate for the ath age of the ith population, denoted by r_{ia}, be defined as the ratio of the number of live births borne by married women at the ath age in a given year in the ith population to the number of married women at the ath age in the ith population. Under some statistical model, r_{ia} has the expectation of R_{ia}, the expected marital fertility rate. In an attempt to parsimoniously describe the variation in the age pattern of human fertility, Coale and Trussell (1974) specified the following model:

$$R_{ia} = n_a \cdot M_i \cdot \exp(m_i \cdot v_a), \tag{1.1}$$

where n_a is the standard age pattern of natural fertility, v_a is the typical age-specific deviation of controlled fertility from natural fertility, and M_i and m_i measure the ith population's fertility level and control. Equation (1.1) states that marital fertility can be modeled as a product of natural fertility and fertility control. The former is represented by $n_a \cdot M_i$; the latter, by $\exp(m_i \cdot v_a)$.

The Coale–Trussell model as expressed in (1.1) has been widely used in demographic research for a number of reasons (see, for example, Hsueh and Anderton 1990; Johnson 1985; Knodel 1977, 1978; Lavely 1986; and Lavely and Freedman 1990). First, the model is justified by the well-understood demographic theory (Coale 1971; Henry 1961) that marital fertility is the combined result of natural fertility and voluntary control of fertility. Second, the two population-specific parameters have intuitive interpretations, with M_i as a scale factor for the underlying level of marital fertility and m_i as an index of the degree of voluntary control. Third, implementation of the model requires only age-specific grouped data; thus the model is useful in demographic studies of historical populations and populations of less-developed countries.

The Coale–Trussell model of Equation (1.1) was initially proposed as a mathematical model to summarize and compare fertility rates. Coale and Trussell (1974) estimated n_a's by taking the age-specific averages of Henry's (1961) 10 natural fertility schedules. For v_a's, Coale and Trussell examined 43 fertility schedules reported in the United Nations's *Demographic Yearbook 1965* (United Nations 1966). Their estimation method was first to set v_1 (i.e., v for the first age interval) to 0, so as to obtain estimates for M_i as r_{i1}/n_1. They next let m_i in Equation (1.1) be 1, let R_{ia} be r_{ia}, and let v_a vary with i. Solving Equation (1.1) for v_{ia} gives

$$v_{ia} = \log(r_{ia}/(M_i \cdot n_a)). \tag{1.2}$$

Coale and Trussell (1974, 1975) calculated the averages of v_{ia} across the 43 schedules and reported the averages as their estimates of v_a's. Setting v_1 to 0 and m_i to 1 normalizes the parameters, because it is not possible to uniquely identify (a) the scales of m_i and v_a or (b) the scale of M_i and the location of v_a (Xie 1991).

Coale and Trussell's (1974) specification of Equation (1.1) is a powerful mathematical model. With a slight but important modification, Broström (1985) and Trussell (1985) turned it into a statistical model by considering observed and expected births (b_{ia} and B_{ia}) instead of observed and expected rates (r_{ia} and R_{ia}), through

$$b_{ia} = T_{ia} \cdot r_{ia} \tag{1.3}$$

and

$$B_{ia} = T_{ia} \cdot R_{ia}, \tag{1.4}$$

where T_{ia} is the total exposure to giving birth in terms of woman-years. Substituting (1.1) into (1.4) and taking the natural logarithm on both sides of the equation yields

$$\log(B_{ia}) = \log(T_{ia} \cdot n_a) + \log(M_i) + m_i \cdot v_a. \tag{1.5}$$

If the n_a and v_a parameters are exactly known, and if births follow an independent Poisson distribution in each age interval of each population, then $\log(M_i)$ and m_i of equation (1.5) can be estimated via maximum likelihood (ML) as the constant and the slope parameters of a log-linear regression

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model with v_a as the regressor. The term $\log(T_{ia} \cdot n_a)$ is included as an independent variable with a known coefficient of unity. In practice, n_a's and v_a's are normally taken from Coale and Trussell (1974, 1975).

The Poisson assumption in the estimation of Equation (1.5) is plausible in the analysis of fertility. First, births are discrete counts. Second, the Poisson distribution implies larger variance at higher fertility (Broström 1985; Rodriguez and Cleland 1988; Wilson, Oeppen, and Pardoe 1988). The advance made by Broström (1985) and Trussell (1985) in transforming the Coale–Trussell method into a Poisson regression model not only facilitated its application to sample data with a conventional estimation procedure but also made statistical inferences possible. But a weakness of the Broström and Trussell approach lies in the assumption that Coale and Trussell's n_a and v_a estimates are exactly true. Wilson et al. (1988) noted that the 10 schedules used by Coale and Trussell (1974) in their computation of the natural fertility standard were sample data. Xie (1990) reanalyzed the same data in the form of births and exposure with Poisson regression models similar to that of Broström (1985) and Trussell (1985). Through a comparison of explicit alternative model specifications, Xie (1990) supported Coale and Trussell's (1974) multiplicative $n_a \cdot M_i$ specification for natural fertility. In addition, Xie (1990) reestimated n_a's through the appropriate log-linear model and found the new estimates preferable to the old estimates of Coale and Trussell.

Xie (1991) further argued that v_a's should also be treated as unknowns and be estimated from sample data. This approach changed Equation (1.5) from a log-linear model to a log-multiplicative model (Clogg 1982; Goodman 1981). The log-multiplicative model for two-way cross-classified tables was first discussed by Goodman (1979) and is sometimes referred to as Goodman's association model II or simply the row and column (RC) model (also see Agresti 1984, 1990 and Goodman 1984). For extensions to cross-table comparisons, see Goodman (1986, pp. 262–266) and Xie (1991, 1992).

2. DATA

In this article we apply Xie's (1991) methods to a data set compiled from the World Fertility Survey (WFS), a group of internationally coordinated surveys of women of reproductive age. The surveys were carried out between 1974 and 1982 in 42 countries, of which we use data from 41. (Iran is excluded because it never completed a country report.) We choose to use these data because of their high quality and comparability. Hundreds of studies have used data from the WFS; Goldman (1985, p. 62) concluded that "for almost all countries, WFS surveys have achieved a better coverage of live births than have previous surveys, censuses, or vital registration systems." For an overall review of WFS sample design and implementation, see Cleland and Scott (1987, chaps. 11–13).

To be consistent with Coale and Trussell (1974), our analysis is restricted to women in six age intervals (20–24, 25–29, 30–34, 35–39, 40–44, and 45–49). All countries except Venezuela interviewed women age 20–49; for the Venezuela sample, the oldest age of women interviewed was 44. We solve this problem by including a dummy variable denoting the missing data for the age group 45–49 in Venezuela in all statistical models. The effect is to ignore the entry for which data are missing while preserving useful information from the Venezuela sample (see Goodman 1968).

In the appendixes of two comparative studies of socioeconomic differentials in fertility, Alam and Casterline (1984) and Ashurst, Balkaran, and Casterline (1984) reported age-specific marital fertility rates (Table A1) and their corresponding woman-years of exposure (Table A2) for a total of 41 populations in the WFS. We ignore the fact that these tabular data were weighted by sampling weights within country for countries that adopted a nonproportionate sample design. For a recent review of alternative solutions to log-linear modeling of contingency tables with weighted counts, see Clogg and Eliason (1988). In the original reports, fertility rates and exposure were disaggregated by the woman's age and education (or another socioeconomic variable). We reconstruct the number of births for each 5-year age interval of each country and each education by country classification. The reconstructed data are presented in the form of rates and exposure at the country level in Tables A.1 and A.2 on pages 982–983.

3. RESULTS

Table 1 reports the goodness-of-fit statistics of models estimated via ML based on (1.4) and (1.5), assuming a Poisson distribution for observed births in each age interval of each population.

There are two panels of parallel models in Table 1. In panel A, the old natural fertility standard of Coale and Trus-

Table 1. Models for Age-Specific Marital Fertility Schedules, World Fertility Survey Country Data

Model	*Description**	L^2	*df*	*D (%)*	*BIC*	X^2
A1	Natural fertility model	6,936.4	204	7.20	4,478.5	7,191.9
A2	Log-linear model	1,366.0	163	2.18	−597.9	1,959.6
A3	Log-multiplicative model	1,037.4	159	1.83	−878.3	1,451.3
B1	Natural fertility model	8,011.6	204	7.77	5,553.8	7,734.3
B2	Log-linear model	735.2	163	1.88	−1,228.7	869.0
B3	Log-multiplicative model	631.1	159	1.74	−1,284.6	754.0

NOTE: L^2 is the log-likelihood ratio chi-squared statistic with the degrees of freedom reported in column *df*. *D* is the index of dissimilarity. BIC = $L^2 - (df)\log(N)$, where *N* is the total number of births. X^2 is the sum of the Pearson chi-squared statistics of all predictions under the appropriate model, with 163 degrees of freedom. All models are based on fertility schedules for 41 countries from the World Fertility Survey (Alam and Casterline 1984; Ashurst et al. 1984). (N = 170,847). In models A1, A2, and A3, Coale and Trussell's (1974) n_a is used. In models B1, B2, and B3, Xie's (1990) n_a is used.

* Besides the description, all models include a dummy variable denoting the sixth age interval (45–49) for Venezuela, where fertility information for women older than 44 was not collected in the survey.

sell (1974) is used. In panel B, the new natural fertility standard of Xie (1990) is used. Both standards are reproduced in Table 2. We choose to apply previously estimated natural fertility schedules to the WFS data rather than to simultaneously estimate n_a's with other parameters, because an earlier draft of this article (Xie and Efron 1991) carried out simultaneous estimation with results similar to those presented here.

In each panel, three models—the natural fertility model, the log-linear model, and the log-multiplicative model—are reported. The natural fertility model assumes no voluntary fertility control for the WFS data; thus it serves as our baseline model. In the log-linear model, v_a's are assumed to be known and are used as a regressor in a log-linear Poisson regression for any population. In the log-multiplicative model, v_a's are treated as unknown parameters to be reestimated with the current data along with the m_i and M_i parameters. Several measures are used to assess the goodness of fit of the models. The log-likelihood ratio statistic L^2 equals -2 times the log-likelihood. Asymptotically, L^2 and the difference in L^2 between two nested models follow the chi-squared distribution (Bishop, Fienberg, and Holland 1975, pp. 125–130). But it is well known that, with large samples, the log-likelihood ratio test is likely to reject a good model. For this reason we also use Schwarz's (1978) Bayesian criterion (BIC), as adapted by Raftery (1986) for contingency table settings:

$$\text{BIC} = L^2 - (df)\log N, \tag{3.1}$$

where L^2 is the log-likelihood ratio statistic, df is the associated degrees of freedom, and N is the sample size. If BIC is negative, then we should accept the null hypothesis that the model fits the data well as compared to the saturated model, which puts no age pattern constraint across populations. When comparing several models, we should select the model with the lowest BIC value. As a purely descriptive measure of goodness of fit, we also use the index of dissimilarity (Shryock and Siegel 1976, p. 131), denoted as D. The index of dissimilarity here can be interpreted as the proportion of misclassified births.

The L^2 statistic is likely to favor the log-multiplicative model because the ML estimation procedure ensures the maximum value of the likelihood and thus the minimum value of L^2 for the model. To give an "honest" assessment of the predictive power of the models, we repeat the same estimation procedure after sequentially deleting one country at a time, and then predict fertility for the deleted country. In the last column of Table 1 we report the total Pearson chi-squared statistic (X^2) under the predictions. Note that

$$X^2 = \sum_{i=1}^{41} \sum_{a=1}^{6} (b_{ia} - B_{ia}^*)^2 / B_{ia}^*, \tag{3.2}$$

where b_{ia} is observed births and B_{ia}^* predicted births from the single-population estimation of Equation (1.5) with v_a's as the regressor in a Poisson regression. For the log-linear model v_a's are taken from Coale and Trussell (1974); for the log-multiplicative model v_a's are estimated from pooled data of the other 40 WFS countries, excluding the ith country. Except for Venezuela, which contributes 3 degrees of freedom, all other countries each contribute 4 degrees of freedom toward the global measure X^2. Thus there are a total of 163 degrees of freedom.

In both panels the natural fertility model fits the data poorly by all goodness-of-fit measures. The log-likelihood ratio chi-squared statistic L^2 is 6,936.4 in model A1 and 8,011.6 in model B1 for 204 degrees of freedom. In contrast the log-linear model fits the data much better. The log-likelihood ratio chi-squared statistic L^2 is 1,366.0 for model A2 and 735.2 for model B2 for 163 degrees of freedom. Nesting models A1 and A2 and B1 and B2, L^2 is drastically reduced by 5,570.4 and 7,276.4 for a modest decrease of 41 degrees of freedom. Obviously, the hypothesis that the populations do not exercise voluntary fertility control can be rejected outright. By the BIC criterion as well, the log-linear model is preferable to the saturated model, for which BIC is 0, and to the natural fertility model, for which BIC is 7,191.9 in panel A and 7,734.3 in panel B.

The log-multiplicative model further improves the goodness of fit of a model. Between models A2 and A3, L^2 is reduced by 328.6 for 4 degrees of freedom. Even when the large sample size (170,847) is considered, the improvement is significant, as the BIC statistic for model A3 is smaller

Table 2. Comparison of Estimated Parameters

	Age					
Source of estimation	*20–24*	*25–29*	*30–34*	*35–39*	*40–44*	*45–49*
Estimates of the n_a parameters[a]						
Coale and Trussell (1974)	.460	.431	.395	.322	.167	.024
Xie (1990)	.460	.436	.392	.333	.199	.043
Estimates of the v_a parameters[b]						
Coale and Trussell (1974)	0	−.279	−.677	−1.042	−1.414	−1.671
Xie (1991)[c]	0	−.320	−.787	−1.216	−1.657	−1.671
Xie (1991)[d]	0	−.228	−.533	−.856	−1.279	−1.671
Model A3, Table 1	0	−.392	−.924	−1.437	−1.671	−.015
Model B3, Table 1	0	−.329	−.713	−1.194	−1.671	−1.082
Averaging method[e]	0	−.335	−.717	−1.186	−1.671	−1.115

[a] Estimates of the n_a parameters are standardized to be .460 at ages 20–24.
[b] Estimates of the v_a parameters are standardized to be 0 at ages 20–24 and to have a minimum of −1.671. See text for explanation.
[c] The estimation was based on Coale and Trussell's (1974) controlled fertility data and their natural fertility standard.
[d] The estimation was based on Coale and Trussell's (1974) controlled fertility data and Xie's (1990) natural fertility standard.
[e] The averaging method is identical to that of Coale and Trussell (1974) now applied to the WFS country data. Xie's (1990) n_a estimates are used.

(−878.3) than that for model A2 (−597.9). The gain in goodness of fit is also significant for panel B with a reduction of 104.1 in L^2 for 4 degrees of freedom and a decrease of 55.9 in BIC. By the index of dissimilarity, we observe that the log-multiplicative model misclassifies lower proportions of births (1.83% in panel A and 1.74% in panel B) as compared to the log-linear model (2.18% in panel A and 1.88% in panel B). Additionally, the X^2 statistic indicates that the log-multiplicative model predicts observed births with smaller errors than does the log-linear model: the contrast is 1,451.3 vs. 1,959.6 in panel A and 754.0 vs. 869.0 in panel B.

Comparing models in panels A and B reveals that the natural fertility standard of Xie (1990) is superior to that of Coale and Trussell (1975). For the same degrees of freedom, the L^2 statistics of models B2 and B3 are much smaller than those of their corresponding models in panel A (models A2 and A3). The same conclusion holds if we use the BIC, D, and X^2 criteria.

Table 2 reports the estimated n_a and v_a parameters from different sources. We normalize the n_a and v_a parameters to facilitate comparison with earlier estimates. As in Xie (1990), n_a's are normalized so that $n_1 = .460$. We set $v_1 = 0$ and $\min(v_a) = -1.671$ to normalize both the location and the scale of v_a. The two conditions come from Coale and Trussell (1974); the first condition ($v_1 = 0$) was their normalization, and the second condition ($v_6 = -1.671$) was their estimate for the last age interval, which for their data showed the greatest deviation from natural fertility.

The $-v_a$ parameter estimated from model B3 rises to a peak at ages 40–44 and then declines at ages 45–49, indicating diminishing deviations of controlled fertility from the natural fertility standard with age after age 45. This contrasts with the old $-v_a$ standard of Coale and Trussell (1974, 1975), which is a monotonically increasing function of age. Is the peaking of $-v_a$ at ages 40–44 due to our treatment of the data in the form of frequencies instead of rates? To answer this question, we also use Coale and Trussell's (1974) method of averaging fertility rates across the 41 populations with Xie's (1990) n_a estimates. The results are reported in the last row of Table 2. The general pattern of the $-v_a$ estimates by the averaging method is similar to that from model B3. The curvilinear pattern remains.

We propose the following explanation. With increasing marriage duration, the proportion of married women in a population who have reached their desired number of children and have consequently voluntarily stopped having children increases. This explains the rise, between ages 20–44, in the deviation of fertility rates of a population practicing fertility control from the natural fertility standard. As women approach the end of their reproductive years, their fecundity declines as a result of an increase in sterility (Menken, Trussell, and Larsen 1986). Therefore, the presence of declining fecundity, regardless of fertility control, suppresses natural fertility so that controlled fertility and natural fertility converge for the age group 45–49. In other words, the significance of the difference between natural and controlled fertility diminishes in the later years of reproduction due to the dominant role of sterility in limiting fertility.

These results call for a reinterpretation of the v_a parameter initially defined by Coale and Trussell (1974, p. 188) as expressing "the tendency for older women in populations practicing contraception or abortion to effect particularly large reductions of fertility below the natural level." The statement that older women under a controlled fertility regime exhibit more reductions than younger women below the natural level is only true for women age 20–44.

We graphically view the significance of the preceding discussion in Figure 1, which compares observed and predicted fertility rates for five countries with different m's after rescaling M to be 1. The Hutterites (1921–1930) fertility schedule is chosen because its age pattern typically represents that of natural fertility (Xie 1990). The other four countries—Sudan, Indonesia, Trinidad and Tobago, and Portugal—are chosen because their estimated m's are evenly spaced and sufficiently different (i.e., close to .25, .5, .75, and 1.0). In addition, the four countries are representative of the four major regions of the world; that is, Africa, Asia, the Americas, and Europe. For the predicted rates, the v_a estimates from model B3 are used. Note that v_1 is normalized to be 0. The five schedules have the same fertility rate (.460) at ages 20–24 because we set M to the common standard of unity. The five curves for predicted rates first diverge and then converge with age. The ratio between any two adjacent curves at any age is determined by the v_a parameter. The age pattern of estimated v_a's indicates that the ratio begins from 1 at ages 20–24, gradually increases to a maximum at ages 40–44, and then declines thereafter.

4. MODEL CRITICISM

Models reported in Table 1 assume within-country homogeneity. We realize that this cannot be true. Past research has shown significant differentials in fertility control by women's socioeconomic characteristics (Alam and Casterline 1984; Ashurst et al. 1984; Lavely and Freedman 1990). In particular it has been found that women's education increases fertility control. In Table 3 we demonstrate the utility of the log-multiplicative approach in studying within-country variations. The data are rearranged into the 160 education-by-country subpopulations. The six (of which four are free)

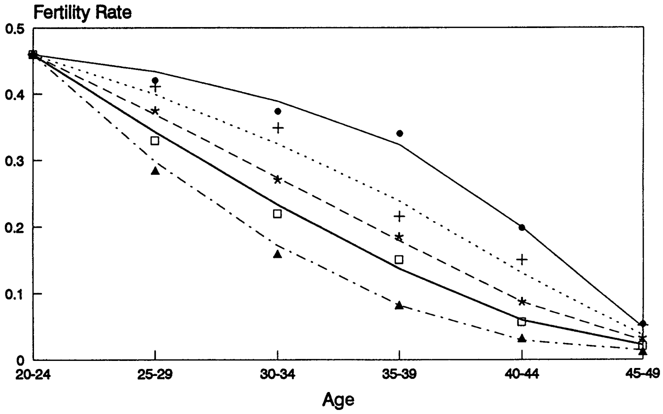

Figure 1. Age Patterns of Marital Fertility, Observed and Predicted. ●, Hutterites, obs.; +, Sudan, obs.; ★, Indonesia, obs.; □, Trinidad/Tobago, obs.; ▲, Portugal, obs.; —, Hutterites, pre.; - -, Sudan, pre.; — —, Indonesia, pre.; —, Trinidad/Tobago, pre.; — -, Portugal, pre.

population-invariant v_a parameters are simultaneously estimated along with the 160 m parameters measuring fertility control and 160 M parameters measuring fertility levels. Note that subpopulations may be formed along many meaningful dimensions such as time, region, ethnicity, residence, and their combinations. A useful extension of this log-multiplicative approach is the reestimation of v_a's for a single country if many subpopulations of the country are compared. This makes the common but inferior practice of borrowing v_a's estimated from other countries to study within-country variations in fertility control (see, for example, Lavely and Freedman 1990) unnecessary.

As the substantive findings of this research, we report the m estimates in Table 3. Note that the scale of the m parameters depends on our particular normalization of v_a; that is, $\min(v_a) = -1.671$. Because there is some arbitrariness in setting $\min(v_a)$ to a constant, the absolute magnitudes of the m estimates are not truly comparable across models. But the relative magnitudes of m's have definite demographic meanings within each model: The greater the m parameter, the greater the fertility control. For populations or subpopulations governed by natural fertility, the estimated m's should be close to 0 within sampling errors. Thus the entries in Table 3 measure the extent to which a population or subpopulation voluntarily limits marital fertility. As expected, the estimated m's are high for countries that either have achieved high levels of economic development (such as Korea) or have implemented successful family planning programs (such as Sri Lanka). The estimated m's are low for most African countries and many other less-developed countries. Additionally, the expected positive effect of education on fertility control generally holds for most of the countries.

The Poisson assumption for the models in Table 1 implies the homogeneity of the risk of birth within age-by-population classifications. As shown, the homogeneity assumption is severely violated with the current data; this violation could lead to overdispersion (McCullagh and Nelder 1989, pp. 198–200). To check for model misspecification error, we standardize residuals from model B3 into Pearson residuals,

$$p_{ia} = (b_{ia} - B_{ia})/B_{ia}^{1/2}, \qquad (4.1)$$

where b_{ia} and B_{ia} are observed and predicted births. We then plot p against $B^{1/2}$ for all cells, as shown in Figure 2. We observe a modest amount of overdispersion for observations with $B^{1/2}$ over 40 and marked overdispersion for observations with $B^{1/2}$ under 10. Because the residual points for different age groups are distinguished with different symbols in Figure 2, it is easy to observe that there are seven outlying countries (with p greater than 4) in the last (45–49) age group. Although this result is consistent with Coale and Trussell's (1978, p. 204) observation that "the fertility rate for women age 45–49 was a conspicuous outlier," we suspect that these outliers could also be caused by age misreporting. The seven outlying countries, all economically less developed, are as follows (with the Pearson residual in parenthesis): Nigeria (10.23), Ghana (7.67), Kenya (6.90), Ivory Coast (5.44), Cameroon (4.77), Haiti (4.60), and Benin (4.04). It is also worth mentioning that five of the seven countries are in West Africa, signalling either a different age pattern or a measurement problem in this region. We note that more elaborate versions of model B3 for treatment of the outliers yield a better fit to the observed data but yield v estimates similar to those discussed earlier. For example, the conservative approach of specifying unique v_6 parameters for the seven countries significantly improves the goodness of fit (L^2 = 414.8 for 152 degrees of freedom, BIC = −1,416.5). The estimated v's from the modified model are 0, −.329, −.714, −1.196, −1.671, and −1.251.

Table 3. Estimated m Parameters by Education and Country

Country	Stratified by education[a] 0 Yrs	1–3 Yrs	4–6 Yrs	7+ Yrs	Total (Model B3)
Africa					
Benin	.180	−.063	.470	.496	.183
Cameroon	.103	.193	.209	.525	.194
Egypt	.466	.777	.841	1.168	.582
Ghana	−.048	.090	.011	.221	.000
Ivory Coast	.056	.029	.498	.201	.074
Kenya	−.026	−.149	.056	.412	.050
Lesotho	.346	.047	.170	.214	.186
Mauritania[b]	.191	.196	—	—	.184
Morocco	.307	.509	.362	.197	.297
Nigeria	.180	.264	.136	.628	.232
Senegal	.188	−.180	.381	.955	.193
Sudan	.222	.322	.600	.452	.255
Tunisia	.280	1.019	.759	1.038	.299
Americas					
Colombia	.407	.367	.669	.959	.453
Costa Rica	.148	.393	.761	.997	.540
Dominican Republic	.281	.327	.771	.994	.410
Ecuador	.163	.185	.484	1.154	.317
Guyana	.923	.733	.887	.628	.681
Haiti	.031	.158	.285	.329	.052
Jamaica	.209	.614	.401	.520	.457
Mexico	.239	.334	.618	.908	.373
Panama	.245	.553	.838	1.138	.702
Paraguay	.132	.326	.570	.701	.384
Peru	.114	.396	.632	.962	.334
Trinidad and Tobago	.758	.206	.751	.791	.721
Venezuela	.378	.322	.842	1.093	.543
Asia and the Pacific					
Bangladesh	.292	.419	.321	.240	.299
Fiji	.936	.653	.571	.857	.742
Indonesia	.496	.139	.339	.984	.495
Jordan	.214	.500	.790	.816	.298
Korea, Republic of	.685	.763	1.076	1.287	.888
Malaysia	.608	.726	.760	.989	.667
Nepal	.155	.047	.335	1.710	.153
Pakistan	.264	.235	.441	1.241	.278
Philippines	.174	.257	.315	.659	.392
Sri Lanka	.482	.520	.629	.732	.588
Syria	.112	.736	.652	.663	.175
Thailand	.227	.381	.378	1.065	.361
Turkey	.631	1.062	1.272	1.590	.712
Yemen AR[b]	−.056	−.042	—	—	−.049
Europe					
Portugal	.768	1.246	1.636	1.426	1.149

[a] The L^2 statistic is 1,150.9 with 627 degrees of freedom. Estimated v_a's are (0, −.267, −.638, −1.140, −1.671, and −1.252).

[b] Only two categories (no schooling vs. some schooling) are distinguished in the questionnaire: the label "1–3 Yrs" should be read as "some schooling" for Mauritania and Yemen.

5. CONCLUSION

Criticism of the Coale–Trussell method has mounted in recent years. Aside from conceptual issues pertaining to

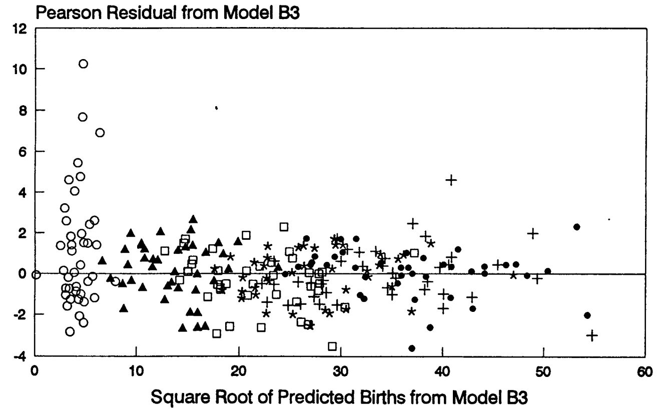

Figure 2. Pearson Residuals vs.Square Root of Predicted Births. ●, Age 20–24; +, Age 25–29; ★, Age 30–34; □, Age 35–59; ▲, Age 40–44; ○, Age 45–49.

marriage duration (Page 1977), parity (David, Mroz, Sanderson, Wachter, and Weir 1988; Pullum 1990), and childlessness (Ewbank 1989), critics have also pointed out that empirically the Coale–Trussell method does not fit observed data well. For example, Anderson and Silver (1992) noted large disparities between observed fertility rates and those predicted by the traditional Coale–Trussell method for many modern populations.

Our study attempts to salvage the Coale–Trussell method by transforming it into a statistical model. The original formulation as expressed in Equation (1.1) is preserved, but with v_a's modified to unknowns and subject to reestimation, as age patterns of human fertility in populations of interest change. In this way, the Coale–Trussell method is made flexible, generalizable, and interpretable. In accordance with earlier work in this area, we also advocate such applications to fertility data in the form of births and exposure to giving birth. Furthermore, we recommend the simultaneous reestimation of the deviation standard (v_a's) along with the estimation of the m and M parameters in a log-multiplicative framework.

For researchers who study a single population or subpopulation, we recommend the log-linear Poisson regression model of Broström (1985) and Trussell (1985). But we advise future users to use the new v_a estimates from model B3 in place of those of Coale and Trussell (1974). Different from earlier estimates, the $-v_a$ standard estimated from this study does not increase monotonically with age, as Coale and Trussell (1974) argued that it should. Rather, $-v_a$ increases and then decreases. We attribute this departure to a difference in data source. Whereas earlier estimations (Coale and Trussell 1974; Xie 1991) were based on data reported in *Demographic Yearbook 1965* (United Nations 1966), our study utilizes more recent data from the WFS.

APPENDIX: DATA ON AGE PATTERNS OF FERTILITY IN 41 COUNTRIES

Table A1. Observed Age-Specific Marital Fertility Rates from the World Fertility Survey

	Age						
Country	20–24	25–29	30–34	35–39	40–44	45–49	TMFR*
Africa							
Benin	.3567	.3403	.2899	.2038	.1062	.0576	6.772
Cameroon	.3238	.2879	.2325	.1676	.1180	.0505	5.902
Egypt	.3912	.3308	.2375	.1467	.0563	.0187	5.906
Ghana	.3081	.2923	.2648	.2028	.1417	.0763	6.430
Ivory Coast	.3448	.3189	.2585	.2163	.1424	.0678	6.743
Kenya	.3989	.3720	.3090	.2551	.1700	.0730	7.890
Lesotho	.3230	.2840	.2566	.1830	.0983	.0377	5.913
Mauritania	.3467	.3323	.2725	.2002	.1069	.0502	6.544
Morocco	.3906	.3440	.2420	.1960	.1113	.0346	6.592
Nigeria	.3258	.2840	.2395	.1569	.1091	.0749	5.951
Senegal	.3562	.3485	.2757	.2079	.1126	.0367	6.688
Sudan	.3630	.3242	.2751	.1701	.1182	.0403	6.454
Tunisia	.4377	.3632	.2765	.2107	.1213	.0417	7.255
Americas							
Colombia	.3810	.2786	.2035	.1627	.0786	.0287	5.665
Costa Rica	.3147	.2198	.1612	.1129	.0629	.0120	4.418
Dominican Republic	.4049	.3098	.2647	.1952	.0703	.0182	6.315
Ecuador	.3733	.3228	.2348	.1864	.1021	.0191	6.192
Guyana	.3722	.2662	.1972	.1228	.0491	.0082	5.079
Haiti	.3336	.3193	.2626	.2121	.1411	.0702	6.695
Jamaica	.3205	.2521	.1973	.1298	.0670	.0169	4.918
Mexico	.4370	.3532	.2902	.2015	.0986	.0263	7.034
Panama	.3617	.2560	.1758	.1228	.0445	.0100	4.854
Paraguay	.3714	.3007	.2349	.1688	.0860	.0190	5.904
Peru	.4269	.3372	.2820	.1955	.1082	.0332	6.915
Trinidad and Tobago	.2637	.1889	.1260	.0861	.0326	.0116	3.544
Venezuela	.3769	.2722	.2087	.1420	.0698	—	5.348
Asia and the Pacific							
Bangladesh	.3262	.2744	.2348	.1661	.0814	.0173	5.501
Fiji	.3539	.2500	.1767	.0992	.0498	.0093	4.694
Indonesia	.3179	.2592	.1873	.1279	.0603	.0224	4.874
Jordan	.4902	.4142	.3553	.2605	.1137	.0235	8.287
							5.578

(continued)

Table A.1. (continued)

Country	Age 20–24	25–29	30–34	35–39	40–44	45–49	TMFR*
Malaysia	.4120	.3050	.2223	.1468	.0477	.0133	5.736
Nepal	.3126	.3075	.2608	.1859	.1036	.0368	6.036
Pakistan	.3500	.3461	.2777	.1995	.0854	.0130	6.358
Philippines	.4372	.3302	.2698	.1953	.0956	.0243	6.762
Sri Lanka	.3554	.2916	.2110	.1353	.0523	.0145	5.300
Syria	.4581	.4023	.3403	.2623	.1488	.0508	8.313
Thailand	.3592	.2720	.2069	.1749	.0813	.0235	5.589
Turkey	.3294	.2526	.1666	.0991	.0485	.0026	4.493
Yemen AR	.3957	.3731	.3578	.2515	.2150	.0810	8.370
Europe							
Portugal	.2881	.1783	.0999	.0517	.0202	.0076	3.229

NOTE: Data are reconstructed from fertility data cross-classified by age and education in Alam and Casterline (1984) and Ashurst et al. (1984).

* TMFR = total marital fertility rate. Some of the TMFR's reported in Ashurst, Balkaran, and Casterline (1984, p. 15) are incorrectly labeled "ages 20–49" when they actually refer to "ages 15–49."

Table A.2. Women-Years of Exposure for Fertility Rates in Table A1

Country	Age 20–24	25–29	30–34	35–39	40–44	45–49	Total
Africa							
Benin	3,764	3,385	2,419	2,029	1,380	540	13,517
Cameroon	6,050	5,517	3,909	3,910	2,366	810	22,562
Egypt	7,262	7,517	6,603	5,292	4,171	1,344	32,189
Ghana	4,450	4,210	3,234	3,068	2,092	749	17,803
Ivory Coast	4,712	3,683	2,942	2,492	1,817	594	16,240
Kenya	5,639	5,593	4,206	3,437	2,506	1,153	22,534
Lesotho	3,231	2,721	2,152	1,874	1,651	499	12,128
Mauritania	2,837	2,929	1,886	1,588	983	351	10,574
Morocco	3,331	3,036	2,735	2,787	2,119	871	14,879
Nigeria	7,815	8,534	5,412	4,784	2,433	941	29,919
Senegal	2,814	2,660	2,313	2,151	1,545	465	11,948
Sudan	3,442	3,218	2,657	1,861	1,053	242	12,473
Tunisia	3,120	3,425	3,178	3,410	2,878	997	17,008
Americas							
Colombia	2,493	2,646	2,318	1,999	1,583	624	11,663
Costa Rica	2,398	2,632	2,394	2,014	1,710	751	11,899
Dominican Republic	1,831	1,571	1,444	1,203	882	386	7,317
Ecuador	3,620	3,721	3,323	2,653	2,086	843	16,246
Guyana	3,134	2,698	2,402	2,038	1,774	624	12,670
Haiti	1,806	1,764	1,502	1,262	1,014	377	7,725
Jamaica	2,107	1,916	1,590	1,524	1,307	594	9,038
Mexico	4,936	5,066	4,624	3,956	3,052	1,192	22,826
Panama	2,455	3,080	2,563	1,975	1,590	603	12,266
Paraguay	2,235	2,182	2,176	1,923	1,595	739	10,850
Peru	4,145	4,560	4,090	3,847	3,308	1,367	21,317
Trinidad and Tobago	2,849	2,845	2,558	2,064	1,656	711	12,683
Venezuela	2,497	2,354	1,966	1,710	689	0	9,216
Asia and the Pacific							
Bangladesh	5,966	4,266	3,348	2,611	2,299	742	19,232
Fiji	4,185	4,764	4,002	3,200	2,515	856	19,522
Indonesia	7,335	6,421	6,435	5,678	4,170	1,542	31,581
Jordan	2,990	3,458	2,689	2,501	1,778	559	13,975
Korea, Republic of	3,053	5,246	5,227	4,399	3,300	1,216	22,441
Malaysia	4,463	5,136	5,314	4,451	3,590	1,268	24,222
Nepal	5,671	5,000	3,492	3,370	2,263	811	20,607
Pakistan	4,012	4,231	3,284	2,775	2,478	895	17,675
Philippines	6,764	8,597	8,168	7,299	6,044	2,450	39,322
Sri Lanka	4,545	5,838	5,592	4,872	4,269	1,923	27,039
Syria	3,669	3,648	3,125	2,926	2,447	971	16,786
Thailand	2,908	3,060	2,860	2,675	2,327	936	14,766
Turkey	3,910	3,637	3,114	3,137	2,665	843	17,306
Yemen AR	2,518	2,113	1,500	1,062	824	221	8,238
Europe							
Portugal	2,692	4,042	4,470	4,373	4,430	1,975	21,982

NOTE: Data are reconstructed from fertility data cross-classified by age and education in Alam and Casterline (1984) and Ashurst et al. (1984).

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