THE LOG-MULTIPLICATIVE LAYER EFFECT MODEL FOR COMPARING MOBILITY TABLES*

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I propose the log-multiplicative layer effect model for comparing mobility tables. The model constrains cross-table variation in the origin-destination association to be the log-multiplicative product of a common association pattern and a table-specific parameter. Like Yamaguchi's (1987) uniform layer effect model, the log-multiplicative layer effect model provides one-parameter tests and thus facilitates analysis of the difference in "vertical mobility" between two mobility tables. Compared to the uniform layer effect model, the log-multiplicative layer effect model is far more flexible in specifying the origin-destination association. Virtually all two-way mobility models can be incorporated into the log-multiplicative layer effect model while retaining their usual interpretability. All that is required is that the tables being compared have a common pattern for the origin-destination association. Properties of the new model are demonstrated using three data sets previously analyzed in comparative mobility research. The same methodology can be generalized to the analysis of multiple two-way contingency tables if the two-way association of primary interest is specified to follow a common pattern, albeit with different levels, across the tables.

In the interest of testing the difference in "vertical mobility" with a single parameter, a useful model for comparing mobility tables has been proposed by Yamaguchi (1987). Referred to here as "the uniform layer effect model," the model is characterized by the use of a single parameter describing the uniform difference in the origin and destination association between a pair of mobility tables. The uniform layer effect model is attractive in comparative research on mobility for its parsimony and interpretability (Wong 1990). However, there are three associated disadvantages. First, the model implicitly assumes that the categories of origin and destination are correctly ordered and uniformly distanced for the three-way interaction among origin, destination, and table. Second, it does not allow the flexibility found in Hauser's (1978, 1979) levels model.[1] Third, it cannot be used to compare Goodman's (1979) row and column effects association model II (*RC* model) across tables, because this would lead to structurally distinct models for different tables (Yamaguchi 1987, p. 484, fn. 1).

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[1] As will become clear, it is possible to combine the uniform layer effect model with Hauser's levels model, although such combinations can be difficult to interpret. For whatever reason, Yamaguchi (1987, p. 484) did not consider Hauser's levels model.

ability. All that is required is that the tables being compared have a common pattern for the origin-destination association. Properties of the new model are demonstrated using three data sets previously analyzed in comparative mobility research: Yamaguchi's (1987) data for the United States, Great Britain, and Japan; Erikson, Goldthorpe, and Portocarero's (1982) data for England, France, and Sweden; and Hazelrigg and Garnier's (1976) data for 16 countries.

MODELS FOR COMPARING MOBILITY TABLES

Let R, C, and L respectively denote the row, column, and layer variables. The observed frequency in an $R \times C \times L$ cross-classified table is denoted by f_{ijk}, and the expected frequency F_{ijk}, where $i = 1, \dots I, j = 1, \dots J$, and $k = 1, \dots K$. I consider the case where $R \times C$ is a square table, i.e., $I = J$, although my results are applicable to the more general case $I \neq J$. Following conventions in mobility research, I let row represent origin, column represent destination, and layer represent tables or groups (usually for nations or different time periods). The saturated model describing the three-way table can be expressed as

$$F_{ijk} = \tau \, \tau_i^R \, \tau_j^C \, \tau_k^L \, \tau_{ij}^{RC} \, \tau_{ik}^{RL} \, \tau_{jk}^{CL} \, \tau_{ijk}^{RCL}, \qquad (1)$$

where the τ parameters are subject to the normal "ANOVA-like" normalization constraint that they multiply to 1 along all appropriate dimensions. With the normalization, τ represents the (unweighted) grand mean; τ_i^R, τ_j^C, and τ_k^L represent respectively the marginal effects of R, C, and L; τ_{ij}^{RC}, τ_{ik}^{RL}, and τ_{jk}^{CL} represent respectively the two-way interactions between R and C, R and L, and C and L; and τ_{ijk}^{RCL} represents the three-way interaction among R, C, and L. The saturated model is not very interesting because in most cases it is not parsimonious. Later, I discuss unsaturated models that describe mobility patterns with fewer theoretically interpretable parameters but that nonetheless fit observed data well. In all the models presented, τ, τ_i^R, τ_j^C, τ_k^L, τ_{ik}^{RL}, and τ_{jk}^{CL} parameters are always included so that the marginal distributions of R and C given L are fitted exactly. Goodman (1970, 1981a) and Clogg (1982a) labelled this situation "conditional association" as distinguished from "partial association," which does not fit the R - L and C - L interactions exactly. As with all "conditional association" models, the log-multiplicative layer effect model is interesting insofar as it constrains the τ_{ij}^{RC} and τ_{ijk}^{RCL} parameters. These parameters together measure the extent to which R (origin) and C (destination) are associated in a given layer (table) and the variation of their association across different layers (tables).

The Uniform Layer Effect Model

The uniform layer effect model specifies that the three-way interaction parameter τ_{ijk}^{RCL} be modelled by $\exp(ij\beta_k)$, where i and j are used as interval variables, and β_k is the table-specific parameter describing the strength of the R and C association for the kth table. β_k's are subject to the normalization constraint $\sum \beta_k = 0$ in order to yield a total of K - 1 nonredundant comparisons.[2] As pointed out by Goodman (1969), the two-way association of a given $I \times J$ table can be fully described by the odds-ratios of $(I - 1)(J - 1)$ 2×2 subtables of adjacent row and column categories. Let $\theta_{ij|k}$ $(i = 1, \dots I - 1; j = 1, \dots J - 1)$ denote the conditional odds-ratio for the subtable containing cells in rows i and $i + 1$ and columns j and $j + 1$ for the kth table. Under the uniform layer effect model (omitting superscripts RC for τ_{ij}^{RC}), it is easy to prove

$$\begin{aligned} \theta_{ij|k} &= F_{ijk} F_{(i+1)(j+1)k} / [F_{(i+1)jk} F_{i(j+1)k}] \\ &= \tau_{ij} \, \tau_{(i+1)(j+1)} / [\tau_{(i+1)j} \tau_{i(j+1)}] \exp[ij\beta_k \\ &\quad + (i+1)(j+1)\beta_k - (i+1)j\beta_k - i(j+1)\beta_k] \\ &= \theta_{ij} \exp(\beta_k), \end{aligned} \qquad (2)$$

where θ_{ij} is determined only by the two-way τ_{ij}^{RC} parameters. The unconditional odds-ratio θ_{ij} can be viewed as the overall odds-ratio for the R and C association ignoring L. Note that the estimation of β is affected by the ordering of the origin and destination categories as i and j enter the model as interval variables in $\exp(ij\beta_k)$. Taking the natural logarithm on both sides of equation 2 gives[3]

$$\log(\theta_{ij|k}) = \log(\theta_{ij}) + \beta_k. \qquad (3)$$

Equation 3 shows that in the uniform layer effect model, log-odds-ratios measuring the origin-destination association for any table can be decom-

[2] My normalization for β here is different from that of Yamaguchi (1987), who used dummy variables to make comparisons to a baseline table. The difference is trivial because a choice between the two alternatives is always arbitrary.

[3] Wong's (1990) equation 2 incorrectly contains (ij) for the second quantity.

posed into two additive quantities. The first quantity, $\log(\theta_{ij})$, is the average origin-destination association for all tables concerned; various specifications for $\log(\theta_{ij})$ will be discussed later. The second quantity, β_k, is table-specific but invariant across origin and destination. If β_k is negative, the R and C association for the kth table is uniformly smaller than average. Conversely, if β_k is positive, the R and C association for the kth table is uniformly greater than average. The homogeneity of the R and C association across the tables implies that all β_k's should be zero due to the normalization $\sum \beta_k = 0$, as the only constant that sums to zero is zero itself.

Modelling τ_{ijk}^{RCL} is meaningful only when the association between R (origin) and C (destination) is present, i.e., $\tau_{ij}^{RC} \neq 1$. In other words, the R, C, and L three-way interaction is considered only when the two-way interactions are nontrivial so that the hierarchical structure of the τ parameters is retained.

The Log-Multiplicative Layer Effect Model

I propose the log-multiplicative layer effect model for comparing mobility tables by letting the origin-destination association vary log-multiplicatively with tables. Specifically, I assume

$$F_{ijk} = \tau\, \tau_i^R\, \tau_j^C\, \tau_k^L\, \tau_{ik}^{RL}\, \tau_{jk}^{CL} \exp(\psi_{ij}\, \phi_k), \qquad (4)$$

where ψ_{ij}'s describe the origin-destination association, and ϕ_k's indicate the table-specific deviations in the association. Note the one-to-one correspondence between $\exp(\psi_{ij})$ in equation 4 and τ_{ij}^{RC} in equation 1. Analogous to equations 2 and 3, the conditional log-odds-ratio for the kth table can be written as

$$\begin{aligned}\log(\theta_{ij|k}) &= (\psi_{ij} + \psi_{(i+1)(j+1)} - \psi_{(i+1)j} - \psi_{i(j+1)})\phi_k \\ &= \log\{\tau_{ij}\, \tau_{(i+1)(j+1)} / [\tau_{(i+1)j}\, \tau_{i(j+1)}]\}\phi_k \\ &= \log(\theta_{ij})\, \phi_k . \qquad (5)\end{aligned}$$

That is, the log-multiplicative layer effect model specifies the RC two-way and RCL three-way interactions as the log-multiplicative product of two things: the overall origin-destination two-way association and a deviation parameter for the kth table. The model is a generalization of Goodman's (1979) row and column effects association model II, or simply the RC log-multiplicative model (Goodman 1981b, 1981c; Clogg 1982b), for two-way tables. It is also a special case of a general class of models briefly introduced by Goodman for multidimensional contingency tables in an excellent exposition on the relationship between correspondence analysis and loglinear analysis (Goodman 1986, pp. 262–66). Parameters ψ and ϕ can be seen as latent scales of ordinal variables: ψ's explicate the origin-destination association pattern, whereas ϕ's represent the origin-destination association levels for the different tables being compared.

Being latent, the ψ_{ij} and ϕ_k parameters need to be normalized for identification purposes. For convenience, I first normalize ψ_{ij}'s to be identifiable for any two-way table and then normalize the scale of ϕ_k so that

$$\sum \phi_k^2 = 1.$$

Such rescaling does not change the model, as the scales of ψ_{ij} and ϕ_k cannot be jointly determined:

$$\psi_{ij}\, \phi_k = \psi_{ij} c\, \phi_k / c,$$

where c can be any constant. In the next section, I discuss various ways to normalize the location of the ψ_{ij} parameters for different origin-destination specifications. Normalizing the location of ϕ_k rather than that of ψ_{ij} would be improper because there are no marginal effects corresponding to ψ_{ij}'s in the log-multiplicative layer effect model.

As in the case of the two-way log-multiplicative model (Goodman 1979; Clogg 1982b), the researcher is only interested in the ratios of relative differences in ϕ_k's because the absolute magnitudes of ϕ_k's reflect the particular normalization rule. When ϕ_k's are estimated to be similar, the strength of the origin-destination association is said to be homogeneous across the tables.[4] A ϕ_k larger than average means that the origin-destination association for the kth table is greater than average if the sign of ψ_{ij} is in the direction of a positive association.

Modelling the Origin-Destination Association

Without further constraint, equation 4 simply reparameterizes the full two-way association between origin and destination. While that may occasionally be useful (e.g., Sobel, Hout, and Dun-

[4] Standard errors of the estimates can be obtained using the jackknife method (Clogg, Shockey, and Eliason 1990). However, this complication can often be avoided in practice because hypothesis testing can be conducted with nested models using a χ^2 test.

can 1985), the real pay-off to equation 4 comes with constraints that are placed on ψ_{ij} in accord with the variety of models available for mobility tables (Hout 1983). I briefly review some of the models but interested readers should consult other sources (Hauser 1978, 1979, 1984; Goodman 1978, 1984; Duncan 1979; Agresti 1990). When the origin and destination variables measure relative social standing only, ordering of the categories of the two variables is unambiguous. However, most common classifications (e.g., Blau and Duncan 1967) are multidimensional, mixing social standing with other attributes. The first four of the six models discussed below take advantage of this prior information and treat origin and destination as ordinal variables, and these models were first presented as a unified class of models by Goodman (1979).[5] The following models are used:

(1) Row effect model (R) specifies

$$\tau_{ij} = \exp(j\mu_i) \text{ or } \psi_{ij} = j\mu_i,$$

where μ_i is called the row score with the constraint $\sum \mu_i = 0$. The model requires the correct ordering and equal distances of the destination (C) categories. With the same notation as in equations 3 and 5, the average log-odds-ratio for this model is

$$\log(\theta_{ij}) = \mu_{i+1} - \mu_i,$$

which is the distance between the row scores.

(2) Column effect model (C) specifies

$$\tau_{ij} = \exp(i\nu_j) \text{ or } \psi_{ij} = i\nu_j,$$

where ν_j is called the column score with the constraint $\sum \nu_j = 0$. The model requires the correct ordering and equal distances of the origin (R) categories. The average log-odds-ratio for this model is

$$\log(\theta_{ij}) = \nu_{j+1} - \nu_j,$$

which is the distance between the column scores.

(3) Row and column effects model I ($R + C$) specifies

$$\tau_{ij} = \exp(j\mu_i + i\nu_j) \text{ or } \psi_{ij} = j\mu_i + i\nu_j,$$

where μ_i and ν_j are respectively row and column scores. I constrain the scores so that $\mu_I = \nu_J = \nu_{J-1} = 0$.[6] The model requires the correct ordering of both the origin (R) and the destination (C) categories. The average log-odds-ratio for this model is

$$\log(\theta_{ij}) = (\mu_{i+1} - \mu_i) + (\nu_{j+1} - \nu_j),$$

which is the sum of the distances between the row scores and between the column scores.

(4) Row and column effects model II (RC) specifies

$$\tau_{ij} = \exp(\mu_i \nu_j) \text{ or } \psi_{ij} = \mu_i \nu_j,$$

where μ_i and ν_j are respectively row and column scores with the constraints that $\sum \mu_i = 0$, $\sum \nu_j = 0$, and $\sum \nu_j^2 = 1$. The model does not require the correct ordering of either the origin (R) or the destination (C) categories. The estimation of the scores (μ_i's and ν_j's) reveals the ordering of categories implicit in the model. The average log-odds-ratio for this model is

$$\log(\theta_{ij}) = (\mu_{i+1} - \mu_i)(\nu_{j+1} - \nu_j),$$

which is the product of the distances between the row scores and between the column scores.

(5) Developed from Goodman's (1972, sec. 3 and 4) definition and investigation of a general class of multiplicative models, Hauser's (1978, 1979) levels model maps cells in the R and C two-way classification into levels. The levels are flexible in form, representing collections of cells with similar degrees of the origin-destination association. The levels can be derived either empirically or a priori theoretically. Even though legitimate concerns have been raised concerning the interpretability of such models in mobility research (Pontinen 1982; Hout 1983, pp. 37–51), the levels model is important because it is very general and flexible. Many special models, such as the quasi-independence model, the quasi-symmetry model, and the full-interaction model, can be reparameterized as levels models. Once we understand how the levels model works within the new log-multiplicative comparison framework, our knowledge can be readily extended to many other models (e.g., Goodman 1986, 1991; Hout, Duncan, and Sobel 1987; Becker 1990; Yamaguchi 1990a, 1990b). For an assessment of

[5] The existence of ordering does not necessarily mean that the categories are correctly ordered. Three of the four models require the correct ordering while one (RC) does not.

[6] Three constraints are required. Here a convenient (yet arbitrary) set of constraints is chosen. The same explanation applies to the next (RC) model.

the levels model and its usefulness in comparative research, see Clogg and Shockey (1984).

The levels model does not assume an ordering of either the origin (R) or the destination (C) categories. For a model with H levels, only H - 1ψ parameters are identifiable. I normalize them so that

$$\sum \psi_h = 0, \text{ for } h \in (i,j) \text{ and } h = 1, \ldots H.$$

(6) Finally, the full two-way interaction model (FI) places no restriction on τ_{ij}. As in Hauser's levels model, the FI model does not assume an ordering of either the origin (R) or the destination (C) categories. There are $(I - 1)(J - 1)$ nonredundant $\log(\theta_{ij})$'s. I conveniently treat the full-interaction model as a levels model with $H = (I - 1)(J - 1) + 1$; for normalization, all cells in the first row or the first column are grouped into the first level, with remaining cells treated as unique levels.

Within the log-multiplicative comparison framework, all unknown parameters of the above models are simultaneously estimated with ϕ_k's. Because the parameters pertaining to the two-way origin-destination association, i.e., μ_i, ν_j, and ψ_h, are log-multiplicative to ϕ_k, an iterative maximum-likelihood procedure is generally required with two exceptions: (a) when the origin-destination specification is very simple such as the levels model with $H = 2$ or models R, C, R + C, RC with $I = J = 2$; and (b) when only two tables are compared ($K = 2$) and the origin-destination specification is not RC. The iterative procedure alternately treats one set of estimates (or initial values) as known in updating the other set of estimates (or initial values) until they stabilize. In my experience, convergence is reasonably fast on a personal computer.

REANALYSIS OF YAMAGUCHI'S (1987) DATA FOR THE UNITED STATES, GREAT BRITAIN, AND JAPAN

To demonstrate the usefulness of the log-multiplicative layer effect model, I first reanalyze the data with which Yamaguchi (1987) introduced the uniform layer effect model. The data set comprises 5 × 5 intergenerational mobility tables for the United States, Great Britain, and Japan. Readers can reproduce the results reported here from counts in Yamaguchi (1987).

For ease of comparison to Yamaguchi's results, my reanalysis is restricted to off-diagonal cells. Excluding diagonal cells is often desirable because the origin-destination association can be dominated by diagonal cells representing inheritance. However, excluding diagonal cells necessarily precludes a comparative analysis of them and thus may obscure the results on how the mobility regime varies across tables. I address this weakness in the second and third examples below, where I show that the new model being presented can be easily extended to the treatment of diagonal cells in the form of Hauser's levels model. Here, the interest centers on replicating Yamaguchi's results.

Table 1 displays the results of various models applied to Yamaguchi's data. The null association model (NA) is the baseline model. Five model specifications (R, C, R + C, RC, and FI) are used to describe the two-way origin-destination association. Each specification (except RC) is modified by three subscripts: o stands for cross-national homogeneity; u stands for cross-nationally uniform comparison (u-comparison) under the uniform layer effect model; and x stands for cross-nationally log-multiplicative comparison (x-comparison) under the log-multiplicative layer effect model. The goodness-of-fit of each model is assessed by the log-likelihood ratio chi-square statistic (L^2) along with its degrees of freedom and p-value and by the BIC statistic.[7] BIC is a Bayesian statistic proposed by Raftery (1986) for large samples: $BIC = L^2 - (df)\log N$, where L^2 is the log-likelihood ratio statistic, df is the associated degrees of freedom, and N is the sample size. The rule is to select the model with the lowest BIC value. When BIC is negative, the null hypothesis is preferred relative to the saturated model. Normalized comparison parameters (ϕ's) from the log-multiplicative layer effect models are reported in the last three columns.

As shown in equations 3 and 5, both the uniform layer effect models and the log-multiplicative layer effect models permit a one-degree-of-freedom test for each comparison. For K tables, there are K - 1 nonredundant comparisons. In the current example, two degrees of freedom are used to estimate the comparison parameters; thus the models with the u and x subscripts have two fewer degrees of freedom than their corresponding models with the o subscript. By the log-likeli-

[7] The reader is cautioned that the BIC statistic has not been formally proved for log-multiplicative models. In a private communication, however, Professor Raftery strongly confirms my conjecture that BIC is a valid test for log-multiplicative models (see Raftery 1988).

Table 1. Goodness-of-Fit Results of Models Applied to Off-Diagonal Cells of Yamaguchi's Social Mobility Tables: United States, Great Britain, and Japan

Model	Description	L^2	*df*	*p*	*BIC*	ϕ_1 (United States)	ϕ_2 (Great Britain)	ϕ_3 (Japan)
NA	Null association between *R* and *C*, given *L*	1,336.20	33	.000	997	—	—	—
R_o	Cross-nationally homogeneous row effect association	155.97	29	.000	-142	—	—	—
R_u	Cross-nationally uniform row effect association	147.61	27	.000	-130	—	—	—
R_x	Cross-nationally log-multiplicative row effect association	147.53	27	.000	-130	.6232	.6346	.4570
C_o	Cross-nationally homogeneous column effect association	67.74	29	.000	-230	—	—	—
C_u	Cross-nationally uniform column effect association	60.28	27	.000	-217	—	—	—
C_x	Cross-nationally log-multiplicative column effect association	58.80	27	.000	-219	.5980	.6562	.4601
$(R+C)_o$	Cross-nationally homogeneous row and column effects association I	38.80	26	.051	-228	—	—	—
$(R+C)_u$	Cross-nationally uniform row and column effects association I	33.26	24	.099	-213	—	—	—
$(R+C)_x$	Cross-nationally log-multiplicative row and column effects association I	33.03	24	.103	-213	.6073	.6326	.4806
RC_o	Cross-nationally homogeneous row and column effects association II	37.72	26	.064	-229	—	—	—
RC_x	Cross-nationally log-multiplicative row and column effects association II	32.12	24	.124	-214	.6080	.6311	.4817
FI_o	Cross-nationally homogeneous full two-way *R* and *C* interaction	36.22	22	.029	-190	—	—	—
FI_u	Cross-nationally uniform full two-way *R* and *C* interaction	30.71	20	.059	-175	—	—	—
FI_x	Cross-nationally log-multiplicative full two-way *R* and *C* interaction	30.94	20	.056	-174	.6064	.6305	.4845

Note: L^2 is the log-likelihood ratio chi-square statistic with the degrees of freedom reported in column *df* and the *p*-value in column *p*. $BIC = L^2\text{-}(df)\log(N)$, where *N* is the total number of observations (28,887). The ϕ parameters are normalized so that $\sum \phi_k^2 = 1$.

hood ratio test, most models do not fit the data satisfactorily. Part of the reason for this is the large sample size (28,887). According to the *BIC* statistic, all models except *NA* are preferred to the saturated model. Since the *u*-comparison specification and the *x*-comparison specification are not nested, the choice between the two cannot be made by a conventional chi-square test. It is possible, however, to compare the models with the *BIC* statistic (Raftery 1988). For all but one case (*FI*), the *x*-comparison model yields smaller L^2 and *BIC* statistics than does the corresponding *u*-comparison model. For reasons given later, model *FI* is not preferred for this data set. On the basis of goodness-of-fit statistics, I conclude that the *x*-comparison models are generally better than the *u*-comparison models.

One serious shortcoming of the *u*-comparison is its inability to compare the *RC* specification across nations. Indeed, in a footnote discussing the *RC* specification for cross-national comparison, Yamaguchi (1987, p. 484) effectively proposed model RC_x. That is, the log-multiplicative layer effect model was partially anticipated by Yamaguchi. As Yamaguchi recognized, the RC_x model "has a significant advantage because it does not require any prior ordering of row and column categories." As Yamaguchi's paper was mainly concerned with the uniform layer effect model, the RC_x model was not estimated, and the $(R + C)_u$

model was favored. In my reanalysis of the data, not only does the RC_x model fit the data better than the $(R + C)_u$ model, but the estimation of the RC_x model also suggests that the $(R + C)_u$ model is misspecified. The estimated column scores under the RC_x model do not form a monotonic scale as they would under the correct prior ordering of the column categories.[8] For various models (including log-multiplicative models) to examine asymmetries of this kind, see Yamaguchi (1990b). Note also that the correct ordering of the row and column categories is not required for the RC_x model as it is for the $(R + C)_u$ model.

The RC_x model can be found elsewhere in the literature, though often in disguised forms. For example, it is a special case of Clogg's (1982a) "heterogeneous row and column effects" model with some reparameterization and constraints. It is also a special case of Goodman's (1986, p. 263) model that describes the three-factor interaction in multiplicative terms. Smith and Garnier (1986, 1987) effectively used the RC_x model in combination with Hauser's levels model, although their log-multiplicative specification for cross-group variation was much simpler since only two groups were compared, i.e., $K = 2$. The multidimensional $RC(M)$ model for multiple two-way tables (Goodman 1986; Becker and Clogg 1989) also contains the RC_x model as a special case with single dimension and homogeneous row and column scores across the tables. In social mobility research, Ganzeboom, Luijkx, and Treiman's (1989) ambitious study of 149 intergenerational mobility tables from 35 countries relied heavily on the RC_x model estimated with the computer package ASSOC. More recently, Yamaguchi and Treiman (1990) extended the RC_x specification to the modelling of asymmetries and their cross-national variations.

Erikson and Goldthorpe (1991) criticized Ganzeboom, Luijkx, and Treiman's (1989) application of the RC_x model. A major component of Erikson and Goldthorpe's critique was their observation that the cross-national comparison parameters were "estimated under highly parsimonious (scaled association) models which, according to conventional criteria, are far from fitting the total data array and also many of the mobility tables for individual nations" (pp. 36–37). This critique led Erikson and Goldthorpe to a model that is essentially equivalent to FI_x.[9]

The full-interaction (FI) specification yields an L^2 of 30.71 for u-comparison and 30.94 for x-comparison, the only case where the u-comparison fit is slightly better than that of the x-comparison. However, the full-interaction specification should not be accepted over others, as it is not parsimonious. Nesting models RC_x and FI_x gives an increase of 1.18 in L^2 for 4 degrees of freedom, clearly insignificant. In conclusion, RC_x is the preferred model. Model RC_x has a good interpretation: Variables R, C, and L are treated as ordinal variables and enter the model log-multiplicatively in a symmetrical fashion. The estimated latent scales of R and C establish the general pattern of the origin-destination association, whereas the estimated latent scale of L determines the level of the association for different nations. However, this interpretation hinges on the specification assumption that the row and column scores are homogeneous across tables.

It is important to emphasize that the x-comparison does not change the statistical model assumed for the origin-destination association in the absence of the comparison. Instead, the x-comparison constrains the origin-destination association to vary log-multiplicatively across different groups. It is intermediate between the restrictive case in which all groups have the same origin-destination association and the unconstrained case in which all groups have different parameters for the origin-destination association. In some ways, the log-multiplicative layer effect model is highly restrictive — not only are all groups assumed to follow the same structure for the association parameters, the relative magnitudes of the parameters within groups are also assumed to be constant. It is across groups that the association parameters are allowed to vary by a common factor. Therefore, the log-multiplicative layer effect models, insofar as they fit data, are ideal for comparing the general levels of mobility across groups.

The estimated comparison parameters (ϕ's) confirm Yamaguchi's (1987) observation that the

[8] I thank David Grusky for drawing this to my attention. The normalized estimates of the row and column scores are respectively (-1.4719, -.9045, .1036, .9226, 1.3502) and (-.6177, -.2888, .3587, .6319, -.0841).

[9] Erikson and Goldthorpe's work is apparently an independent effort to address the same problem, albeit from a more theoretical angle, as the present paper. Their paper was presented in November 1991, a few months after an earlier version of this paper was presented at the annual meeting of the American Sociological Association in Cincinnati, Ohio (August 1991). I received a copy of their paper from Professor Erikson in early December 1991.

level of the origin-destination association is similar in the United States and Great Britain and is weaker in Japan than in the other two nations.[10] Lacking appropriate measures, Yamaguchi borrowed the estimated association parameter from the uniform association model for the other models and concluded that "the extent of association between fathers' and sons' statuses found in the off-diagonal elements of the mobility structure is about 20–30 percent less for Japan than for the other two nations" (p. 488).

Yamaguchi's calculation was the best approximation available to him, for the u-comparison does not permit a direct calculation of the relative magnitude of the differences. From equation 3, comparing nations 1 and 2 gives

$$[\log(\theta_{ij|2}) - \log(\theta_{ij|1})]/\log(\theta_{ij|1}) = (\beta_2 - \beta_1)/[\log(\theta_{ij}) + \beta_1]. \quad (6)$$

Because $\log(\theta_{ij})$ is not a single number, equation 6 cannot be further simplified. For the x-comparison, however, the relative magnitude of the differences can be derived from equation 5:

$$\begin{aligned}[\log(\theta_{ij|2}) - \log(\theta_{ij|1})]/\log(\theta_{ij|1}) &= [\log(\theta_{ij})(\phi_2 - \phi_1)]/[\log(\theta_{ij})\,\phi_1] \\ &= (\phi_2 - \phi_1)/\phi_1. \quad (7)\end{aligned}$$

Using the formula and the estimated ϕ parameters for model RC_x, the relative differences are calculated as follows: The origin-destination association is 20.8 percent less in Japan than in the United States and 23.7 percent less in Japan than in Great Britain. These numbers were calculated net of diagonal cells and are very close to those reported by Yamaguchi.

PATTERNS AND AMOUNTS OF MOBILITY: HAUSER'S LEVELS MODEL FOR ENGLAND, FRANCE, AND SWEDEN

A central thesis in comparative research on social mobility is that of Lipset and Zetterberg (1959, p. 13) that "the overall pattern of social mobility appears to be much the same in the industrial societies of various Western countries." This thesis, in its original form, has been empirically invalidated (Broom and Jones 1969a, 1969b; Erikson, Goldthorpe, and Portocarero 1979). The main explanation for the variation in *observed* mobility is that industrialized countries may historically have been at different stages of economic development in the recent past and thus exhibited different occupational structures.

The negative evidence prompted Featherman, Jones, and Hauser's (1975, p. 339) modification that industrial societies "may have similar patterns of *circulation* mobility" net of differing structural mobility. Note that Featherman, Jones, and Hauser proposed their hypothesis in the framework of log-linear models: Structural mobility is represented by parameters for origin and destination marginal distributions whereas circulation mobility is represented by parameters for the origin and destination interactions.[11] More explicitly, Hauser and Grusky (1988, p. 725) restated that the pattern of circulation mobility should refer to "the odds ratios in mobility classifications."

It may not be an overstatement to say that nothing of sociological interest can be truly invariant across countries. What is at issue is the magnitude and the regularity of such variations. If the magnitude of the cross-national variation in circulation mobility is relatively small and apparently unexplainable, the researcher may conclude that the Featherman-Jones-Hauser hypothesis is supported. In the past decade, many researchers have come to this conclusion (Erikson, Goldthorpe, and Portocarero 1982; Kerckhoff, Campbell, and Winfield-Laird 1985). As correctly pointed out by Wong (1990, p. 560), previous research employed statistical methods that simultaneously test different hypotheses and thus are not sensitive. Yamaguchi's uniform layer effect model (u-comparison) enabled more sensitive one-degree-of-freedom tests. The log-multiplicative layer effect model (x-comparison) introduced in this paper provides a superior method for this type of analysis.

In the language of log-linear models, common patterns of social mobility mean that the two-way origin-destination association is common to all tables being compared. The least restrictive version of the hypothesis leaves origin and desti-

[10] Whether mobility regime is homogeneous across the three nations is an issue. Indeed, by the *BIC* criterion, the homogeneity hypothesis is accepted over the alternative heterogeneity hypothesis. Part of the explanation is the similarity between the United States and Great Britain (Yamaguchi 1987). However, the purpose of using the data set is mainly to illustrate the new models and compare them to Yamaguchi's models, not necessarily to find the best-fitting model. Thus I do not report the contrast between Japan and the other two nations.

[11] For a critical discussion of structural mobility versus circulation mobility, see Sobel (1983).

Table 2. Erikson, Goldthorpe, and Portocarero's (1982) Levels Matrix for a Model of the Common Pattern of Social Mobility in England, France, and Sweden

Class of Origin		Class of Destination						
		I+II	III	IVa+b	IVc	V/VI	VIIa	VIIb
I+II	Service class	2	3	4	6	5	6	6
III	Routine non-manual employees	3	3	4	6	4	5	6
IVa+b	Petty bourgeoisie	4	4	2	5	5	5	5
IVc	Farmers	6	6	5	1	6	5	2
V/VI	Skilled working class	4	4	5	6	3	4	5
VIIa	Semi- and unskilled working class	5	4	5	5	3	3	5
VIIb	Agricultural workers	6	6	5	3	5	4	1

nation fully interactive. For the sake of parsimony and maximizing statistical power, however, students of comparative social mobility often try to find restrictions on the origin-destination association. Thus, the concept of a common pattern can be methodologically represented by Hauser's (1978, 1979) levels, which can be obtained empirically as a parsimonious description of the origin-destination association.

One of the best known applications of Hauser's levels models to comparative mobility research is Erikson, Goldthorpe, and Portocarero's (1982) analysis of data for England, France, and Sweden. The three mobility tables were originally classified in nine categories and later collapsed into seven categories. As shown in Table 2, they used six levels to describe the 36 origin-destination interaction parameters.[12] The model is not only very parsimonious but also fits the data well. Hauser (1984) replicated this model and compared it with alternative approaches (Hope 1982) to studying the same data set.

Table 3 displays results of six models applied to the data analyzed by Erikson, Goldthorpe, and Portocarero (1982) and later by Hauser (1984). The *NA* (null association) baseline model yields $L^2 = 4{,}860.03$ for 108 degrees of freedom.[13] Model FI_o leaves the origin-destination association free but restricts it to be the same across the three nations; it reduces L^2 to 121.30 from model *NA* for 36 degrees of freedom. The *BIC* statistic becomes -577, indicating a good fit. Allowing the origin-destination association to vary log-multiplicatively across the three nations, model FI_x further improves the goodness-of-fit ($L^2 = 92.14$ for 70 degrees of freedom, $BIC = -587$). Nesting models FI_o and FI_x, the chi-square statistic is 29.16 for 2 degrees of freedom, highly significant. By the *BIC* criterion, model FI_x is also preferred over model FI_o.

Models H_o through H_l are variants of Hauser's levels model whose levels matrix is reproduced in Table 2. The subscript *l* indicates the full interaction between levels and layers (nations). That is, model H_o assumes homogeneity, model H_l heterogeneity, of the levels across the three nations. The H_o model yields an L^2 of 244.34 for 103 degrees of freedom; the H_l model yields an L^2 of 208.50 for 93 degrees of freedom. These statistics specify the lower and upper bounds of the goodness-of-fit of the six-levels model in a comparative framework. Given the large sample (16,297), Erikson, Goldthorpe, and Portocarero (1982) and Hauser (1984) correctly prefer model H_o over model H_l even though, strictly speaking, the chi-square statistic between the two nested models (35.84 for 10 degrees of freedom) is significant. Model H_o has a lower negative value of *BIC* than does model H_l. With the specification that the origin-destination association vary log-multiplicatively cross-nationally, model H_x is between the other two models. By the chi-square statistic, model H_x ($L^2 = 216.37$ with 101 degrees of freedom) fits the data significantly better than model H_o (chi-square = 27.97 for 2 degrees of freedom) and not significantly worse than model H_l (chi-square = 7.87 with 8 degrees of freedom). Thus model H_x is preferable to either model H_o or model H_l. Additionally, model H_x is the best of all the models in Table 3 according to the

[12] Under the independence model, the number of constrained association parameters is (7 - 1)(7 - 1) = 36. Note that the diagonal cells are included.

[13] This number agrees with Hauser's reanalysis but is slightly larger than that reported by Erikson, Goldthorpe, and Portocarero (1982). My reanalysis is based on the data provided by Hauser (1984, Appendix). Apparently, there is a small discrepancy between the original data and those reproduced by Hauser (1984, p. 106, note 7).

Table 3. Goodness-of-Fit Results of Models Applied to Erikson, Goldthorpe, and Portocarero's (1982) Data on Intergenerational Class Mobility in England, France, and Sweden

Model	Description	L^2	*df*	*p*	*BIC*	ϕ_1 (England)	ϕ_2 (France)	ϕ_3 (Sweden)
NA	Null association between *R* and *C*, given *L*	4,860.03	108	.000	3,813	—	—	—
FI_o	Cross-nationally homogeneous full two-way *R* and *C* interaction	121.30	72	.000	-577	—	—	—
FI_x	Cross-nationally log-multiplicative full two-way *R* and *C* interaction	92.14	70	.039	-587	.6167	.6333	.4676
H_o	Cross-nationally homogeneous levels model (Table 2)	244.34	103	.000	-755	—	—	—
H_x	Cross-nationally log-multiplicative levels model (Table 2)	216.37	101	.000	-763	.6127	.6336	.4723
H_l	Cross-nationally heterogeneous levels model (Table 2)	208.50	93	.000	-693	—	—	—

Note: L^2 is the log-likelihood ratio chi-square statistic with the degrees of freedom reported in column *df* and the *p*-value in column *p*. $BIC = L^2 - (df) \log(N)$, where *N* is the total number of observations (16,297). The ϕ parameters are normalized so that $\sum \phi_k^2 = 1$.

BIC criterion. To conclude, I have shown that most of the cross-national variation in model H_l is captured by the more parsimonious H_x model.

Using a more powerful statistical test (nesting H_0 and H_x), I have shown that the three nations differ in the extent to which destination depends on origin. The normalized estimates of the comparison parameters (ϕ's) in Table 3 reveal that the origin-destination association is similar in England and France and weaker in Sweden than in England and France. This result holds for both the full-interaction model and the six-levels model. Therefore, my findings strengthen the tentative conclusion of Erikson, Goldthorpe, and Portocarero (1982) that "Sweden shows greater social fluidity than the other two countries" (p. 26).

The new models for comparing mobility tables introduced in this paper call for a revision of the Featherman-Jones-Hauser hypothesis. Perhaps it is not actual amounts, but patterns, of social mobility that are invariant across all industrial nations. Here patterns of social mobility are redefined as the differentiation in the relative magnitudes of the levels matrix for the origin-destination association within nations, i.e., relative odds-ratios within nations. Patterns of social mobility could be the same when amounts of social mobility differ by a common factor. In the example of England, France, and Sweden, the same six-levels model is fitted to all the nations. Moreover, the relative magnitudes of the six levels within each country are the same. There is, however, a general multiplier that raises or lowers all association parameters for each country. This model conveys well the revised notion that there exists the same pattern, albeit different amounts, of social mobility in the three industrial nations.

COVARIATES OF COMPARISON PARAMETERS: REPLICATING GRUSKY AND HAUSER'S (1984) SIXTEEN-COUNTRY COMPARISON

In the log-multiplicative framework, one comparison parameter (ϕ) is obtained for each mobility table. It would be interesting, then, to examine how the comparison parameter covaries with other factors (also see Ganzeboom, Luijkx, and Treiman 1989). In a study comparing mobility patterns in 16 countries, Grusky and Hauser (1984) attempted to attribute cross-national variations in mobility to four explanatory variables: industrialization, educational enrollment, inequality, and social democracy. The expectation (from Treiman 1970; Hazelrigg and Garnier 1976; Erikson, Goldthorpe, and Portocarero 1982) was that industrialization, education, and social democracy should have negative effects, whereas inequality should have a positive effect, on destination's dependence on origin. Grusky and Hauser's (1984) results, however, were ambiguous. For industrialization, educational enrollment, and social democracy, they found that "the exogenous variables have nonuniform effects on social fluidity" (p. 34). For inequality, they were puzzled

Table 4. Goodness-of-Fit Results of Models Applied to Hazelrigg and Garnier's (1976) Data

Model	Description	L^2	*df*	*p*	*BIC*
Sixteen Countries					
NA	Null association between *R* and *C*, given *L*	42,970	64	.000	42,225
Q_o	Cross-nationally homogeneous quasi-perfect mobility	1,500	61	.000	790
Q_x	Cross-nationally log-multiplicative quasi-perfect mobility	956	46	.000	421
Q_l	Cross-nationally heterogeneous quasi-perfect mobility	150	16	.000	-36
FI_o	Cross-nationally homogeneous full two-way *R* and *C* interaction	1,329	60	.000	631
FI_x	Cross-nationally log-multiplicative full two-way *R* and *C* interaction	822	45	.000	298
Fifteen Countries (Excluding Hungary)					
NA	Null association between *R* and *C*, given L	39,309	60	.000	38,617
Q_o	Cross-nationally homogeneous quasi-perfect mobility	1,065	57	.000	408
Q_x	Cross-nationally log-multiplicative quasi-perfect mobility	527	43	.000	31
Q_l	Cross-nationally heterogeneous quasi-perfect mobility	149	15	.000	-24
FI_o	Cross-nationally homogeneous full two-way *R* and *C* interaction	910	56	.000	264
FI_x	Cross-nationally log-multiplicative full two-way *R* and *C* interaction	409	42	.000	-75

Note: L^2 is the log-likelihood ratio chi-square statistic with the degrees of freedom reported in column *df* and the *p*-value in column *p*. $BIC = L^2 - (df)\log(N)$, where *N* is the total number of observations (113,556 for sixteen countries and 101,505 for fifteen countries).

by its negative effects on the origin-destination association.

I reanalyze the data originally assembled by Hazelrigg and Garnier (1976) and later used by Grusky and Hauser (1984). Table 4 presents results from two sets of models applied to the data. Models in the upper panel are based on data from the 16 countries. Models in the lower panel are based on data from the 15 countries after excluding Hungary. Separating Hungary from other countries was suggested by Grusky and Hauser (1984), who singled out Hungary as a consistent outlier.[14] Within each panel, there are six models: null association (*NA*), cross-nationally homogeneous quasi-perfect mobility (Q_o), cross-nationally log-multiplicative quasi-perfect mobility (Q_x), cross-nationally heterogeneous quasi-perfect mobility (Q_l), cross-nationally homogeneous full two-way *R* and *C* interaction (FI_o), and cross-nationally log-multiplicative full two-way *R* and *C* interaction (FI_x). Diagonal cells are included as distinct levels: *Q* models have a total of four levels, three of which fit the diagonal cells, and *FI* models have a total of five levels.[15]

[14] Grusky and Hauser tried to explain the Hungarian case. For example, they considered grouping all socialist countries.

[15] Four odds-ratios describe the full interaction in a 3 × 3 table: (3 - 1)(3 - 1) = 4.

Each *Q* model is nested within an *FI* model. As before, the subscript *o* denotes models for cross-national homogeneity; *l* for cross-national heterogeneity; and *x* for the log-multiplicative layer effects. The models with the *x* subscript are less restrictive than those with the *o* subscript and more restrictive than those with the *l* subscript.

By the L^2 criterion, none of the models in the upper panel fits the data satisfactorily. Grusky and Hauser (1984) argued, however, that model Q_l "fits extremely well, accounting for 99.7 percent of the association under the baseline model of independence" (p. 24) (L^2 reduced from 42,970 in the *NA* model to 150 in the Q_l model). They did not blindly rely on L^2 statistics because the sample size is very large (113,556). In a comment on Grusky and Hauser's paper, Raftery (1986) provided a formal justification for Grusky and Hauser's "commonsense" conclusion: Model Q_l had a *BIC* statistic of -36 and therefore should be preferred to other models with larger *BIC* statistics including the saturated model, for which *BIC* is 0. By either L^2 or *BIC*, two models of cross-national homogeneity (Q_o, FI_o) and two models of cross-national log-multiplicative effects (Q_x, FI_x) are rejected.

The Hungarian case contributes a significant portion (30 to 50 percent) to the lack of fit of models Q_o, FI_o, Q_x, and FI_x in the upper panel. This is shown by contrasting the upper and lower panels. For just four degrees of freedom, L^2 for model Q_o is reduced from 1,500 in the upper panel to 1,065 in the lower panel. For model FI_x, L^2 is reduced from 822 to 409 for 3 degrees of freedom. The *BIC* statistic of model FI_x drops from 298 to -75.

It is of interest to compare model FI_x to model Q_l. The difference lies in where to attribute variations. Model Q_l has a simpler model for within-nation variation: There is a one-degree-of-freedom quasi-perfect mobility constraint for each nation. The three parameters within each nation, however, are free to vary cross-nationally. In contrast, model FI_x gives four parameters to each nation, which would lead to a perfect prediction of the observed frequencies if there were no cross-national constraints. The log-multiplicative cross-national constraint in model FI_x means that all nations share a common pattern of the four parameters and differ by a multiplier. Here a "common pattern" refers to the relative magnitudes of the *FI* parameters for any given nation. Because models Q_l and FI_x are not nested, there is no firm ground, with a conventional chi-square statistic, on which to make a choice between the two. By the parsimony principle, however, model FI_x is more attractive than model Q_l because the former uses fewer parameters. In the lower panel, for example, model FI_x has 42 degrees of freedom while model Q_l has only 15 degrees of freedom. Furthermore, relative to the saturated model, model FI_x in the lower panel fits better by the *BIC* criterion than does model Q_l. I tentatively conclude that model FI_x in the lower panel is the best model. At the least, model FI_x is no less plausible than model Q_l.

Table 5 gives the estimated measures of social immobility (ϕ's) from four models (Q_x and FI_x, from both panels of Table 4). The four sets of estimates are quite consistent. Table 5 indicates that Sweden has higher immobility than the United States, contrary to my earlier reanalysis of Erikson, Goldthorpe, and Portocarero's (1982) data. This may reflect the poor quality of Hazelrigg and Garnier's 16-country data and/or the limitations of 3 × 3 mobility tables, which lose too much detailed information on within-stratum mobility flows.

The correlation matrix for Grusky and Hauser's four explanatory variables and the four measures of social immobility is displayed in Table 6. The four measures give virtually identical results — the correlation between any two measures of social immobility is between .997 and .999. The correlations among the explanatory variables are in the expected directions. What is surprising, however, is the absence of any evidence bearing on the relationship between the explanatory variables and the measures of social immobility. The correlations between the explanatory variables and the measures of social immobility are almost all zero. The best that could be concluded from the matrix is that industrialization, educational enrollment, social democracy, and inequality do not have systematic effects on circulation mobility.[16] This conclusion is consistent with Grusky and Hauser's (1984) finding that the four explanatory variables do not affect mobility uniformly.

In light of these results, I advance the following hypothesis: Even though industrialized countries differ in the level of circulation mobility, the differences cannot be attributed to either economic development or political processes. The unique histories of different industrialized countries leave the differences in the level of circulation mobility virtually unexplainable.

Table 5. Normalized Measures of Social Immobility (ϕ's) for 16 Countries

Country	Model Q_x	Model FI_x	Model $Q_x{}^\dagger$	Model $FI_x{}^\dagger$
Australia	.2151	.2170	.2215	.2228
Belgium	.2965	.2968	.3059	.3062
France	.2779	.2788	.2875	.2882
Hungary	.2510	.2459	—	—
Italy	.2994	.2995	.3081	.3078
Japan	.2290	.2306	.2394	.2406
Philippines	.2341	.2376	.2437	.2461
Spain	.2963	.2959	.3070	.3062
United States	.2384	.2379	.2429	.2429
West Germany	.2179	.2220	.2269	.2303
West Malaysia	.1941	.1984	.2018	.2051
Yugoslavia	.2230	.2235	.2311	.2310
Denmark	.2827	.2809	.2908	.2887
Finland	.2307	.2265	.2399	.2351
Norway	.1960	.1963	.2025	.2024
Sweden	.2785	.2752	.2849	.2820

Note: The ϕ estimates are normalized so that $\sum \phi_k^2 = 1$.

† Models are based on 15 countries (excluding Hungary).

[16] To be exact, correlation coefficients measure only linear relationships. I checked but failed to find other forms of relationships.

Table 6. Pearson's Correlation Coefficients Between Explanatory Variables and Measures of Social Immobility for 16 Countries

Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) Industrialization	1.000	—	—	—	—	—	—	—
(2) Educational enrollment	.842	1.000	—	—	—	—	—	—
(3) Inequality	-.539	-.618	1.000	—	—	—	—	—
(4) Social democracy	.483	.503	-.362	1.000	—	—	—	—
(5) ϕ scores, Model Q_x	.089	.022	-.072	-.039	1.000	—	—	—
(6) ϕ scores, Model FI_x	.062	.013	-.035	-.044	.997	1.000	—	—
(7) ϕ scores, Model Q_x	.063†	.003†	-.053†	-.039†	.999†	.998†	1.000	—
(8) ϕ scores, Model FI_x	.040†	-.002†	-.032†	-.054†	.997†	.999†	.999†	1.000

† Coefficient based on 15 countries (excluding Hungary).

CONCLUSION

I have proposed the log-multiplicative layer effect model for comparing mobility tables. This model formulates the table-specific origin-destination association as the log-multiplicative product of a common pattern and a table-specific comparison parameter. In this way, the new model retains the desirable property of having a one-degree-of-freedom test per contrast as does the uniform layer effect model (Yamaguchi 1987). The usefulness of the model was demonstrated with applications to three data sets previously analyzed by students of comparative social mobility. Even though the three examples all involved cross-national comparisons, extensions of the models to trend analysis (Hout 1988) require no modification and thus should be straightforward.

The reanalysis of three data sets with the new models sheds new light on Featherman, Jones, and Hauser's (1975) revision of Lipset and Zetterberg's (1959) thesis that circulation mobility in all industrialized nations is much the same after the structurally induced differences in occupational structure are taken into account. I found that industrialized nations can differ in the level of circulation mobility within the new log-multiplicative framework. I offer the following revision of the Featherman-Jones-Hauser hypothesis: What might be constant in all industrialized nations is the "pattern of circulation mobility," which is redefined as the relative magnitudes of association parameters for a given nation. Nonetheless, two characteristics are retained from the earlier hypotheses. First, the pattern of circulation mobility is basically the same for all industrialized nations once the pattern is redefined. Second, the level of circulation mobility among industrialized nations is sociologically "much the same" because the differences cannot be explained by conventional measures of economic development and political programs aimed at reducing social inequality.

The log-multiplicative models can be extended more generally to accommodate multidimensional specifications for the two-way R and C association. This might be necessary either because the model being considered cannot be easily parameterized in a single dimension, e.g., the crossings model (Goodman 1972; Pontinen 1982), or because the researcher wishes to partition ψ_{ij} into different dimensions, as in Hout's (1984) SAT (status, autonomy, and training) model (also see Hope 1982, Hauser 1984). Specifically, $\exp(\psi_{ij}\,\phi_k)$ may take the form:

$$\exp(\sum_{m=1}^{M} \psi_{ij}^{m}\,\phi_K), \qquad (8)$$

where ψ_{ij}^{m} is the mth dimension of the R and C association and should be further parameterized as specific functions of R and C. Note that equation 8 is analogous to the multidimensional $RC(M)$ model (Goodman 1986; Becker and Clogg 1989) for two-way tables restricting either row or column scores to be the same for all dimensions. Even though it could be relaxed, this restriction is often desirable to yield one-degree-of-freedom tests. A different type of extension to multiple dimensions allows the researcher to decompose two-way association parameters into two parts: (a) those that do not vary across tables and (b) those that vary across tables. In other words, one could follow Goodman's (1986, p. 263) suggestion and specify

$$\exp(\psi_{ij}\,\phi_k) = \delta_{ij}\exp(\psi_{ij}^{*}\,\phi_k). \qquad (9)$$

This specification has been used in empirical research. For example, Grusky and Hauser (1984, p. 30), Hauser (1984, p. 101), and Yamaguchi (1987, p. 485) all let a general inheritance parameter vary while constraining specific inheritance parameters to be constant across nations.[17] Smith and Garnier (1986, 1987) used a "hybrid model" that combined homogeneous levels parameters and the usual *RC* association parameters.

The key contribution of this research is the methodological innovation of modelling multiple mobility tables in a log-multiplicative framework. The methodology builds on the assumption that the pattern of the two-way association of primary interest is common for all tables, but that the level of the association differs across tables. The new log-multiplicative models can be easily extended to other areas of comparative research whenever the above assumption is reasonable. For example, Xie (1991) and Xie and Pimentel (1991) used the log-multiplicative models for comparing fertility limitation with Coale and Trussell's (1974) specification that controlled fertility's deviation from natural fertility is the log-multiplicative product of an age pattern and a population-specific control parameter. Clearly, the log-multiplicative approach for comparative research need not be limited to mobility and fertility studies.

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[17] This type of model presents no special problem for estimation and interpretation because there are only two levels (diagonal versus off-diagonal) for ψ_{ij}^*. It is worth noting that with two levels, ψ_{ij}^* can be parameterized by a dummy variable. Clearly there are only K - 1 identifiable parameters in ($\psi_{ij}^*\phi_k$) (assuming redundancy between δ_{ij} and ψ_{ij}^*, otherwise K); and these K - 1 parameters provide one-degree-of-freedom tests. Under this special condition, the log-multiplicative models presented here reduce to simple log-linear models with additive three-way effects (also see Yamaguchi 1987, 1988).

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